THE LAMPSHADE DESIGN MAKING USE OF GEOMETRIC FORMS: THE LAMPSHADE FORM BASED ON FOUR-DIMNSIONAL POLYTOPE WITH TENSEGRITY STRUCTURE

O DESIGN DE ABAJURES COMPOSTOS POR FORMAS GEOMÉTRICAS: A FORMA DE ABAJURES BASEADA EM POLITOPOS TETRADIMENSIONAIS COM ESTRUTURA DE TENSEGRIDADE

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Abstract: Lampshades were developed as wind shield for light source when fire was used for light source. Though contemporary light sources, like LED lamp, fluorescent lamp, and incandescent light, are working with electricity and wind shield is not required for light source, almost all light sources are still covered by lampshade to arrange or control luminous flux from light source. And combination of lampshades and light sources may give distinctive impression to surrounding people. The geometric forms play a significant role in the design of lampshades. In this paper, author introduced the idea of a form by 3D projection of a 4D form including a tensegrity structure, and tried to design a new type of lampshade.

Keywords: lampshade, geometric form, tensegrity, polytope.

Resumo: Os abajures foram desenvolvidos para proteger do vento uma fonte de luz oriunda do fogo. Embora as fontes de luz contemporâneas (como lâmpada LED, lâmpada fluorescente e luz incandescente) funcionem com eletricidade e a proteção contra o vento não seja mais necessária, quase todas ainda são cobertas por um abajur para organizar ou controlar o fluxo luminoso da luz. A combinação de abajures e fontes de luz proporciona uma impressão distinta às pessoas ao redor. As formas geométricas, então, desempenham um papel significativo no design dos abajures. Neste artigo, é introduzida a ideia de uma forma por projeção 3D de uma forma 4D incluindo uma estrutura de tensegridade, e efetua-se a tentativa de projetar um novo tipo de abajur.

**Palavras-chave:** abajur, forma geométrica, tensegridade, politopo.

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1   Introduction

Lampshades were developed as wind shield for light source when fire was used for light source. Though contemporary light sources, like LED lamp, fluorescent lamp and incandescent light, are working with electricity and wind shield is not required for light source, almost all light sources are still covered by lampshade to arrange or control luminous flux from light source. And combination of lampshades and light sources may give distinctive impression to surrounding people.

The luminance distribution of the lampshade surface is affected by the inverse square law of illuminance, which is determined by the distance between the light source and the surface, and the cosine law of illuminance, which is determined by the direction of incident light and the direction of the surface, as shown in Fig. 1.
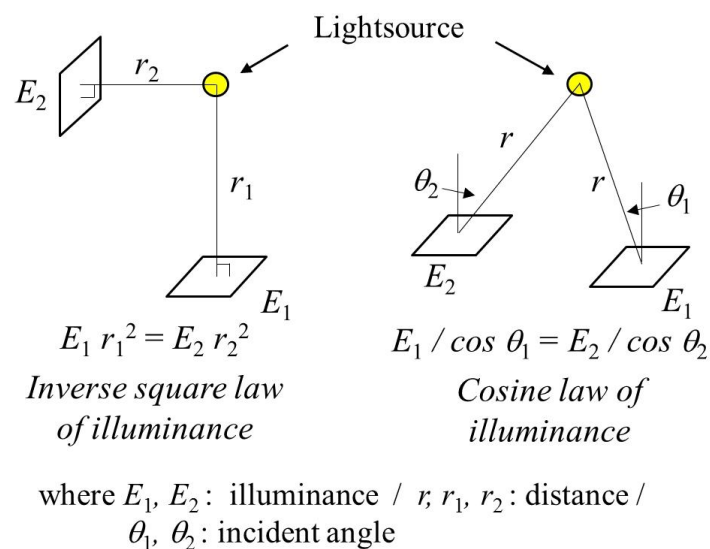
Figure 1 – Basic laws of illuminance distribution

For this reason, the form of the lamp shade has been designed considering the luminance distribution of the surface. For example, LE KLINT, a lampshade manufacturer, fabricates polyhedral lampshades using paper folding technology as shown in Fig. 2 (left). In addition, Louis Poulsen, a lampshade manufacturer as well, fabricates lampshades that use a curved surface based on logarithmic spiral to keep the values of the illuminance on the surface constant as shown in Fig. 2 (right).

In this way, the geometric forms play a significant role in the design of the lampshade. In this paper, author introduced the idea of a shape by 3D projection of a 4D shape including a tensegrity structure, and tried to design a new type of lampshade. Until now, various types of tensegrity structures have been proposed. In recent years, a type

has been proposed in which a plural structures composed of compression materials are used, and the upper end of the lower structure is suspended from the lower end of the upper structure by a tension material to make the upper structure appear to be floating. Though the tensegrity structure itself is not new, there are still various possibilities with the structure. On the other hand, high-dimensional forms including hypercubes cannot be handled as they are in 3D space, however, they can be represented as 3D models or 2D plane shapes using projection, which is the basic technique of descriptive geometry.

**Figure 2 –** The lampshade product by LE KLINT (left) and that by louis poulsen (right)

In this study, the tensegrity structure is applied to a three-dimensional model of an oblique projection of a hypercube, and a tension material is used diagonally in addition to the eight edges connecting the two cubes to form a tensegrity structure. And, finally, diffuse surfaces were attached to the forms, indicating that they could work as lampshades.

2   Lampshade design with geometric forms

As explained in the previous section, various researchers and designers have been working on lampshade design using geometric forms in the past, including the products shown in Fig. 2. In geometrical forms, plane surfaces and curved surfaces are classified as shown in Fig. 3.

As shown in the figure, all surfaces are classified into plane surface and curved surface at first. Then, curved surfaces are classified into four classes from viewpoint of differential geometry. All curved surfaces are classified into ruled surface and double curved surface. Ruled surface can be described as locus of a straight line movement.

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Double curved surface, including sphere and torus, cannot be described such. Double curved surface is not developable and cannot be made from plane surface. Ruled surface is classified into developable surface and warped surface. Developable surface is including conical surface, cylindrical surface and tangent surface, and can be made from plane surface by bending the plane surface. Warped surface is including hyperboloid of revolution, hyperbolic paraboloid and helicoid, is not developable.
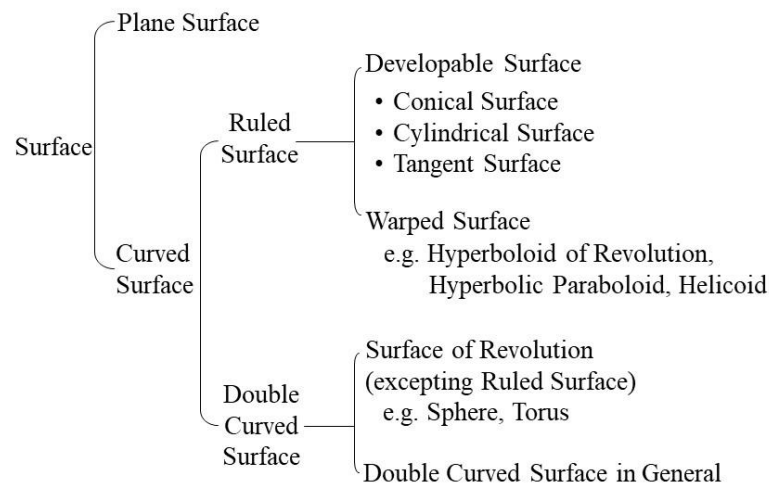
Figure 3 – The classification of geomettric forms

2.1   Lampshade design with plane surfaces

As for plane surfaces, various types of polyhedra have been used in lampshade design as shown in Fig. 4.

Figure 4 – Small stellated dodecahedron used for the lampshade. From the book Abe et. Al (2020, p. 30)

The form called 'PCCP (Pseudo Cylindrical Concave Polyhedral)' shell, Yoshimura pattern, or Diamond pattern is one of the most famous polyhedron used for lampshade design, as shown in Fig. 5 (left).

Figure 5 – The PCCP shell (left) and the PCCC shell (right) used for a lampshade

Yoshimura pointed out similarity between shape of PCCP shell and shape made by buckling of cylindrical shell with compressive power (YOSHIMURA, 1951). Miura named the shape as Pseudo-Cylindrical Concave Polyhedral and surveyed several features of PCCP shell (MIURA, 1969). In case of PCCP structure, each diamond unit is constructed by two plane triangles. Consequently, luminance distribution in the area of the unit is discontinuous at boundary of the two triangles. Suzuki pointed out that similar shape appears with folding method of PCCP shell if the process of valley folding is omitted as shown in Fig. 5 (right) (Suzuki, 2014). And Suzuki named the folding method as Skew Quadrilateral Elastic Folding (hereafter, SQEF) and also named the cylindrical shape constructed by SQEF as Pseudo-Cylindrical Concave Curves (hereafter, PCCC) shell (Suzuki, 2015). In case of PCCC shell, each skew quadrilateral is constructed by continuous curved surface and illuminance distribution is continuous within area of the unit quadrilateral as shown in Fig. 5 (right). From the viewpoint of shape, difference between two shapes is trivial, however, from the viewpoint of illuminance distribution, difference between two shapes is significant. As for the shape of PCCC shell, Suzuki et. al. concluded that exact shape of diamond unit in the shell is foldable but not developable by qualitative analysis making use of 3D scanner (SUZUKI, 2016). And Okamura et al. and Odaka et al. conducted psychological evaluation test for PCCC lampshade respectively (OKAMURA et. al., 2016 and ODAKA et.al., 2016). And Suzuki proposed

the expansion of PCCC shell design method making use of plane tessellation (SUZUKI, 2017, SUZUKI[a], 2018 and SUZUKI[b], 2018) as shown in Fig. 6 and dissection (SUZUKI, 2019) as shown in Fig. 7.


Figure 6 – The expamsion of the PCCC shell. Utilization of a hexagon (left), utilization of Japanese traditional pattern (middle), and utilization of wave shape (right)

Figure 7 – The expamsion of the PCCC shell making use of dissection with Pythagorean Tiling

SUZUKI extended SQEF method to paper folding technique which make use of a nonconvex shape as a horizontal section of lampshades to obtain convex bump texture on the lampshades as shown in Fig. 8 (SUZUKI, 2019).
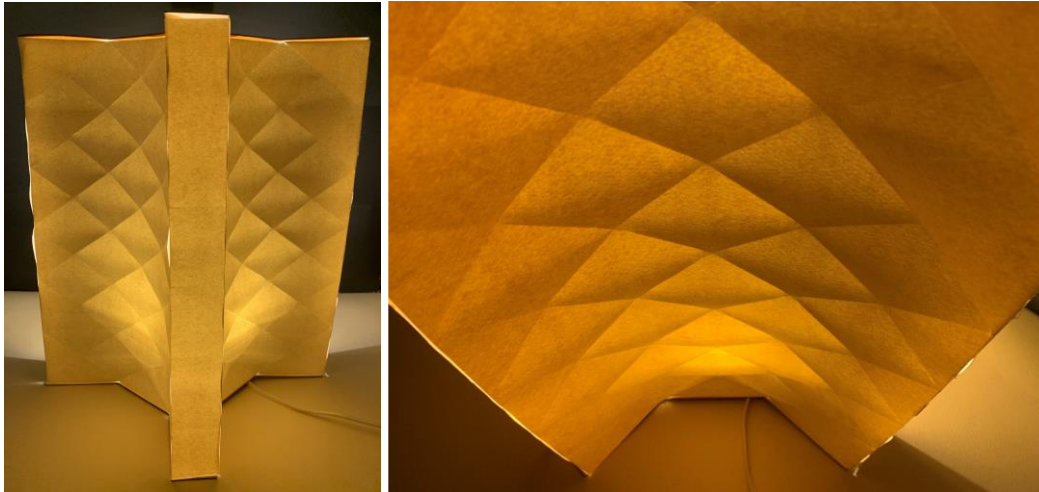
**Figure 8 –** The lampshade with convex bump texture

2.2 Lampshade design with curved surfaces

Manufacturing of developable surfaces is very easy, though designing of the surfaces have much restrictions. Therefore, a lot of researches have been conducted to increase freedom of shape design with developable surfaces. Suzuki proposed method of shape generation with combination of tangent surfaces (SUZUKI, 2010) and Suzuki extended the tangent surface method with hermite curve (SUZUKI, 2012).


Figure 9 – The lampshade making use of tangent surface: designed image (left) and the fabricated model (right)

Suzuki proposed method of shape generation with manipulation of curved line to generate connected developable surfaces (SUZUKI[a], 2014) as shown in Fig. 10, and Suzuki implemented the design method on CG freeware POV-Ray making use of affine transformation and locus diagram (SUZUKI[b], 2014).

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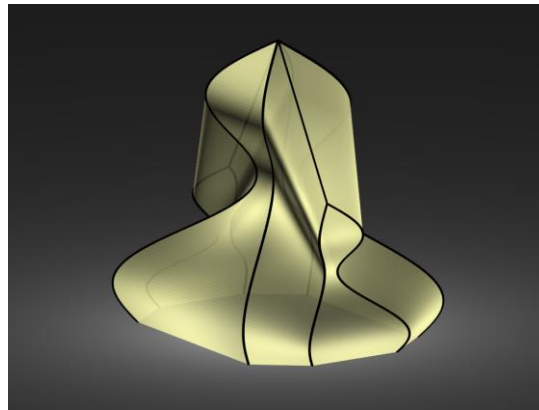
Figure 10 – An example of lampshade designed by the shape generation method with manipulation of curved line to generate connected developable surfaces

Suzuki et al (2015) also introduced the curved line manipulation method into graphic science education for designing and manufacturing of lampshade to evaluate proposed method and developed interface as shown in Fig. 11.
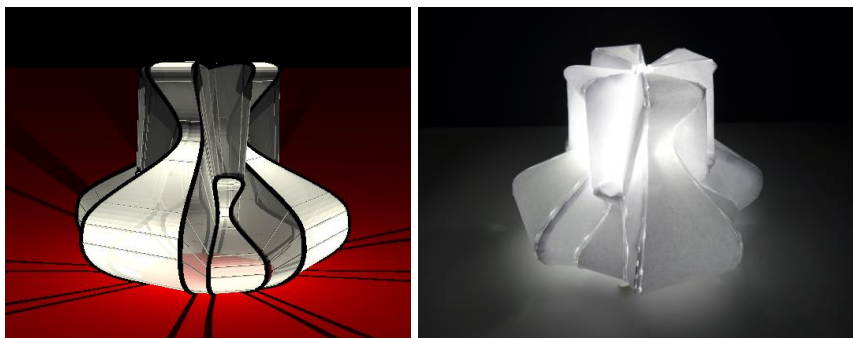

Figure 11 – The submitted work designed by combination of developable surface method: designed CG image (left) and the fabricated model (right)

## 3   Tensegrity structures and projection of 4D forms into a 3D space

3.1   Tensegrity structures

It is said that the first tensegrity structure is the work that sculptor Kenneth Snelson showed to architect/designer Buckminster Fuller in a summer course held in 1948. Fuller named the structure tensegrity, and he himself was trying to apply it to fields such as architectural structures (Miyazaki, 2016). The work, named X-piece, is made up of two X-shaped members made of compression material connected by multiple tension materials, as shown in Fig. 12.

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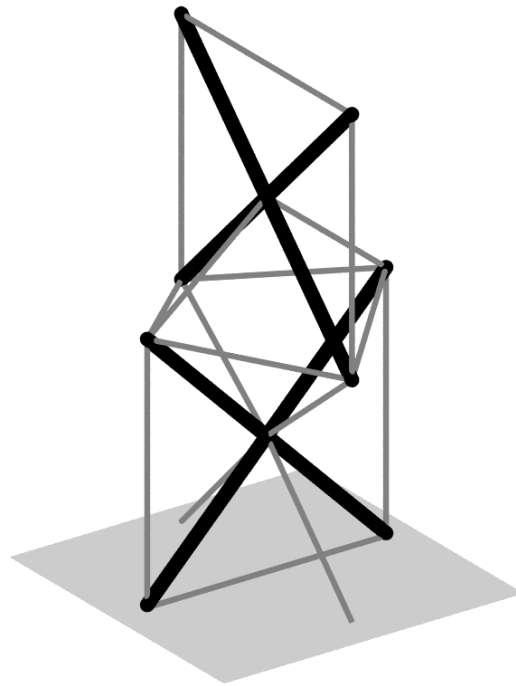
Figure 12 – X-piece by Kenneth Snelson. The black members are the compression
material and the gray materials are the tension material
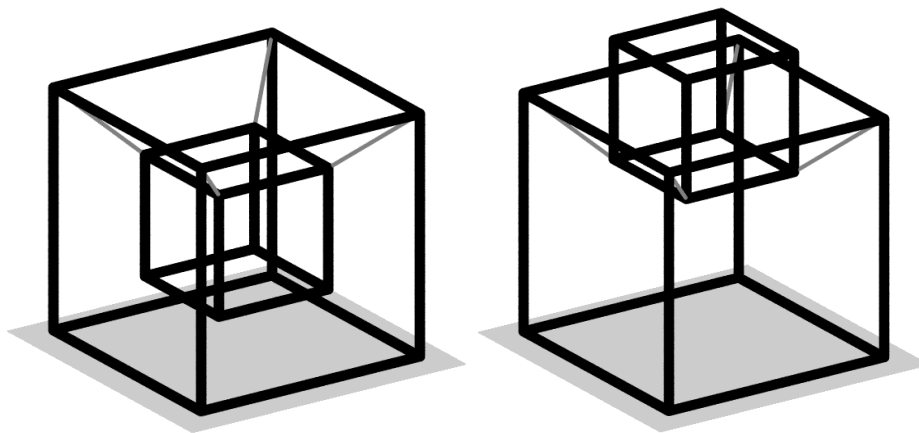
Figure 13 – Shapes that combines two large and small cubes made of compression material
with a tension material. The black members are the compression
material and the gray materials are the tension material.

Here, "compression material" means a material that resists both compression force
and tension force, and "tension material" means a material that resists only tension force.
If tensegrity structure is defined as "a self-supporting structure in which two or more
members composed only of compression material are connected only by tension
material", a form in which a small cube made of a compression material is suspended in
a large cube made of a compression material is also included as shown in Fig. 13 (left),

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though the form does not give the unique floating impression peculiar to the tensegrity structure. Therefore, in this paper, the tensegrity structure is defined as "a self-supporting structure in which two or more members composed only of a compression material are connected only by a tension material, and the upper end of upper member is located higher than the upper end of lower member." Fig. 13 (right) shows a small cube made of compressed material suspended in a large cube made of compressed material as well. However, the upper end of the small cube is located higher that of the large cube and the form meets the definition.

3.2 Projection of 4D forms into a 3D space

The projection of 4D forms into a 3D space has been introduced in various documents (for example, MIYAZAKI, 2020), however, few of them describe the projection in detail by combining graphical explanations and explanations of coordinate transformations. In this section, the explanations are described.

First, consider the projection from 3D forms to a 2D plane by combining graphical explanations and explanations of coordinate transformation. Projections from 3D forms to 2D plane are classified as shown in Fig. 14 (ABE et. al., 2020). If projecting lines are parallel to each other, the projection is classified as parallel projection, and if the lines intersect at one point, the projection is classified as perspective projection.
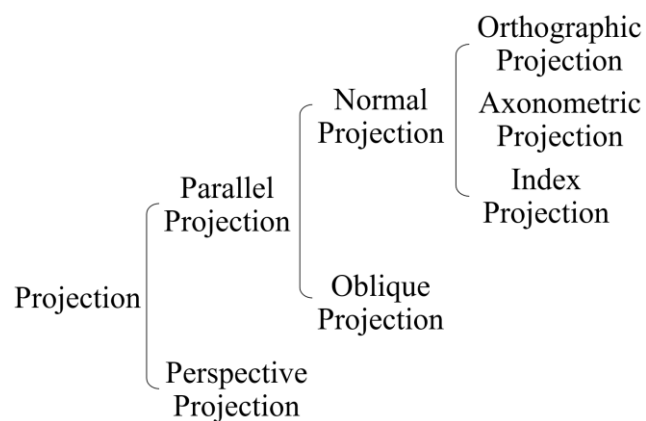

Figure 14 – Classification of projections

Furthermore, in parallel projection, depending on the relationship between the projecting line and the projecting plane, the projection is classified as normal projection when they are crossing vertically, and is classified as oblique projection when they are crossing diagonally. And in normal projection, when the main axis constituting the form is parallel or perpendicular to the projecting line, it is orthographic projection, when it is

not parallel nor perpendicular, it is axonometric projection, and when it is difficult to set the main axis for a form such as a curved surface, the projection is classified as an index projection represented by contour lines. As the classification in normal projection is based on the shape and layout of the 3D forms, and the classification based only on projecting lines and the projecting surface consist of three projections, normal projection, oblique projection, and perspective projection.

Here, let us consider the 3D space by the xyz axis in the left hand system and consider the plane of $z = 0$ as projecting plane. Then, all kinds of projections can be considered as movement from the point $\vec{P}$ (P_x, P_y, P_z) to the point $\vec{P}'$ (P'_x, P'_y, P'_z), where $P'_z = 0$. Then the three projections can be described as Fig. 15.
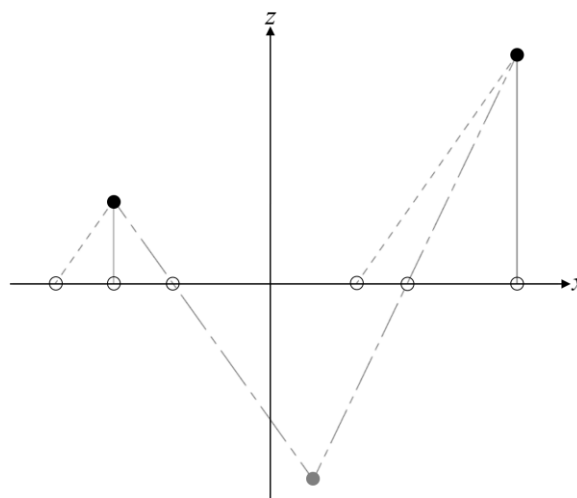
Figure 15 – Movement of three projections in 3D space. The black circles are the points before the projection, the white circles are the points after the projection, the gray circle is the viewpoint, the solid line is corresponding to the normal projection, the dotted line, the oblique projection, and the dashed line, the perspective projection

Then let us consider that the following equation represents all projections.

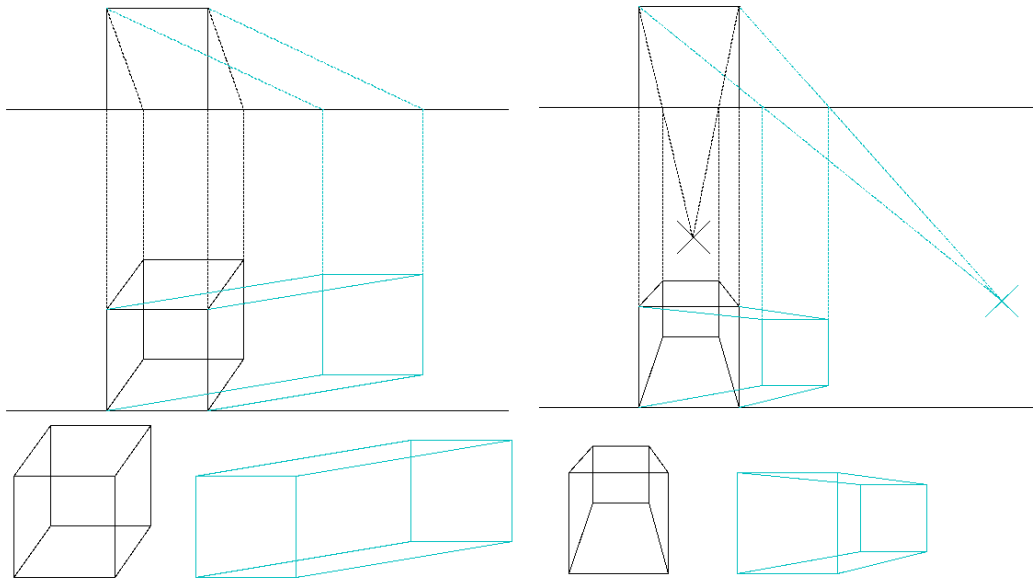$$\overrightarrow{P'} = \vec{P} + \vec{d} \times P_z \tag{1}$$

where,

$$\vec{d} = \begin{pmatrix} d_x \\ d_y \\ -1 \end{pmatrix} \tag{2}$$

It is assumed that the value of P'_z is always 0. In case of the normal projection, $\vec{d}$ and $\overrightarrow{P'}$ are given as follows:

$$\vec{d} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \times P_z = \begin{pmatrix} P_x \\ P_y \\ 0 \end{pmatrix} \tag{4}$$

Figure 16 – Projection of a cube where one of the faces of the cube is in contact with the projecting plane: oblique projection (left) and perspective projection (right)

On the other hand, in oblique projection, the direction of the projecting line is not limited, though the figure drawn by the projection is distorted as the projecting line and the projecting plane become closer to parallel.

$\vec{d}$ and $\overrightarrow{P'}$ are given by the following equation with d_x and d_y as arbitrary

$$\vec{d} = \begin{pmatrix} d_x \\ d_y \\ -1 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} + \begin{pmatrix} d_x \\ d_y \\ -1 \end{pmatrix} \times P_z = \begin{pmatrix} P_x + d_x \times P_z \\ P_y + d_y \times P_z \\ 0 \end{pmatrix} \tag{6}$$

In the case of oblique projection of a cube where one of the faces of the cube is in contact with the projecting plane ($z = 0$), oblique projection of another face of the cube parallel to the projecting plane can exist at any place as shown in Fig. 16 (left). In other words, if two squares of the same size are drawn at arbitrary positions and the corresponding points are connected, the drawing is corresponding to an oblique projection of the cube where one of the faces of the cube is in contact with the projecting plane.

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In the case of perspective projection, $\vec{d}$ and $\overrightarrow{P'}$, which are depending on the position of the point $\vec{P}$ (P_x, P_y, P_z) and the viewpoint $\vec{S}$ (S_x, S_y, S_z), are given as follows:

$$\vec{d} = \frac{\vec{S} - \vec{P}}{P_z - S_z} \tag{7}$$

$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} + \frac{1}{P_z - S_z} \begin{pmatrix} S_x - P_x \\ S_y - P_y \\ S_z - P_z \end{pmatrix} \times P_z \tag{8}$$

In the case of a perspective projection of a cube in which one of the faces is in contact with the projecting plane ($z = 0$), perspective projection of another face of the cube parallel to the projecting plane can exist at any place and in any size as shown in Fig. 16 (right). In other words, if two squares of different sizes are drawn at arbitrary positions and the corresponding points are connected, the drawing is corresponding to a perspective projection of the cube where one of the faces of the cube is in contact with the projecting plane.

It is easy to understand 4D to 3D projections if the above-mentioned relationship is applied to 4D. Let us consider the 4D space by the $xyzw$ axis and consider the hyper plane of $w = 0$ as projecting plane. Then, all kinds of projections can be considered as movement from the point $\vec{P}$ (P_x, P_y, P_z, P_w) to the point $\vec{P'}$ (P'_x, P'_y, P'_z, P'_w) where $P'_w = 0$. In the same way, the viewpoint is $\vec{S}$ (S_x, S_y, S_z, S_w). Then all 4D to 3D projection can be described as follows:

$$\overrightarrow{P'} = \vec{P} + \vec{d} \times P_w \tag{9}$$

where

$$\vec{d} = \begin{pmatrix} d_x \\ d_y \\ d_z \\ -1 \end{pmatrix} \tag{10}$$

It is assumed that the value of P'_w is always 0. In the case of a normal projection, $\vec{d}$ and $\overrightarrow{P'}$ are given as follows:

$$\vec{d} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \\ P'_w \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \\ P_w \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \times P_w = \begin{pmatrix} P_x \\ P_y \\ P_z \\ 0 \end{pmatrix} \tag{12}$$

In case of oblique projection, $\vec{d}$ and $\overrightarrow{P'}$ are given by the following equation with d_x, d_y, and d_z as arbitrary.

$$\vec{d} = \begin{pmatrix} d_x \\ d_y \\ d_z \\ -1 \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \\ P'_w \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \\ P_w \end{pmatrix} + \begin{pmatrix} d_x \\ d_y \\ d_z \\ -1 \end{pmatrix} \times P_w = \begin{pmatrix} P_x + d_x \times P_w \\ P_y + d_y \times P_w \\ P_z + d_z \times P_w \\ 0 \end{pmatrix} \tag{14}$$

In the case of perspective projection, $\vec{d}$ and $\overrightarrow{P'}$ are given as follows:

$$\vec{d} = \frac{\vec{S} - \vec{P}}{P_w - S_w} \tag{15}$$

$$\begin{pmatrix} P'_x \\ P'_y \\ P'_z \\ P'_w \end{pmatrix} = \begin{pmatrix} P_x \\ P_y \\ P_z \\ P_w \end{pmatrix} + \frac{1}{P_w - S_w} \begin{pmatrix} S_x - P_x \\ S_y - P_y \\ S_z - P_z \\ S_w - P_w \end{pmatrix} \times P_w \tag{16}$$

When a cube in the hyper plane ($w = 0$) which edges are parallel to any of the xyz axes is moved parallel to the w axis by the length of the edge of the cube, its locus becomes a hypercube. Let us consider 4D to 3D projection of the hypercube on the projecting surface, hyper surface ($w = 0$). In the case of a normal projection, the cubes before and after the projection are exactly the same. In the case of oblique projection, a cube of the same size as the one before projection is placed at an arbitrary position, and the corresponding points before and after movement are connected as edges to form a hypercube in 3D space. In the case of perspective projection, a cube of any size is placed at an arbitrary position and the corresponding points before and after the movement are connected as edges. Figure 17 (left) shows a model of 3D projection of a hypercube by oblique projection and Figure 17 (right) shows that by perspective projection. These drawings are axonometric projection of the models of 3D projection of a hypercube.

4 The form by 4D to 3D projection of hypercube including tensegrity structure

In this section, the form by 4D to 3D projection of hypercube including tensegrity structure is explained. When a hypercube is projected into a 3D space by oblique projection, two cubes of the same size appear, so these are considered to be the lower and upper parts

composed of compression materials in the tensegrity structure. And a tension material can be used for the edges which are connecting corresponding vertices. However, as shown in Fig. 17 (left), the form cannot be tensegrity structure if the two cubes are completely separated. Also, even if the two cubes intersect, there must be a tension material that suspends lower point of upper member from upper point of lower member. Here, the diagonal of the hypercube is used as the position of the tension material.
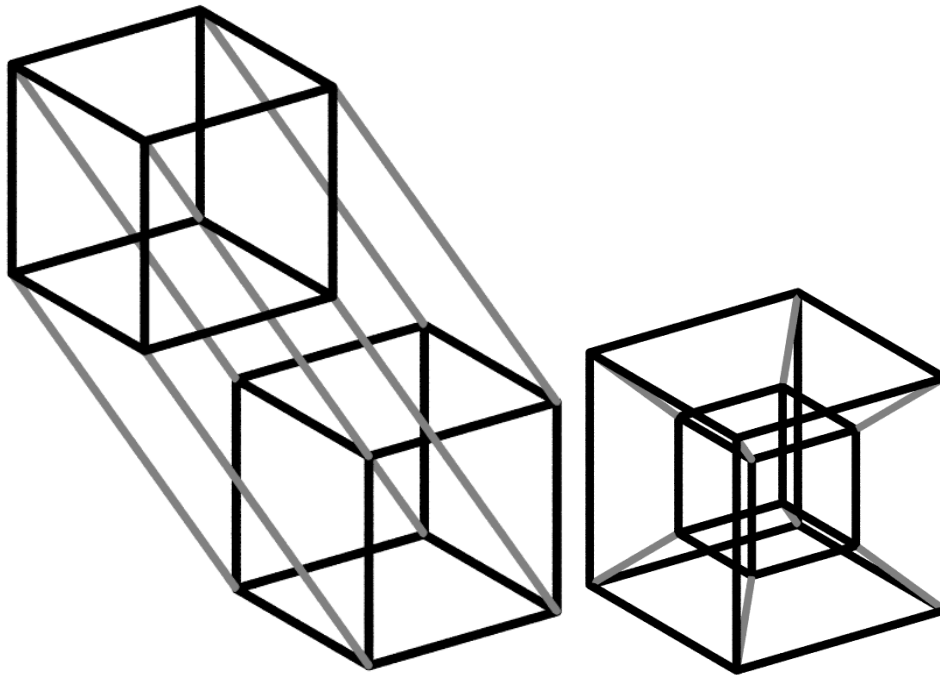

Figure 17 – Models of 3D projection of a hypercube: oblique projection (left) and perspective projection (right)

Since there are 16 vertices in a hypercube, the combination connecting the two points is $120 \, (= 16 \times (16 - 1)/2)$, including 88 diagonal lines and 32 edges (see Fig. 18).

Of these, 64 $(= 8 \times 8)$ are connecting two cubes. Let $\vec{T} \, (T_x, T_y, T_z)$ be the vector between corresponding points of two cubes in 3D projected space and $\vec{D} \, (D_x, D_y, D_z)$ be the vector corresponding to the tension material added at the end to establish this form as tensegrity structure. Considering the balance of power between two cubes, the vector and must meet following conditions:

$$T_x \times D_x \leq 0, \;\; T_y \times D_y \leq 0, \;\; T_z \times D_z \leq 0 \qquad (17)$$

Of the 64 line segments, only one shown in Fig. 19 meets these conditions.
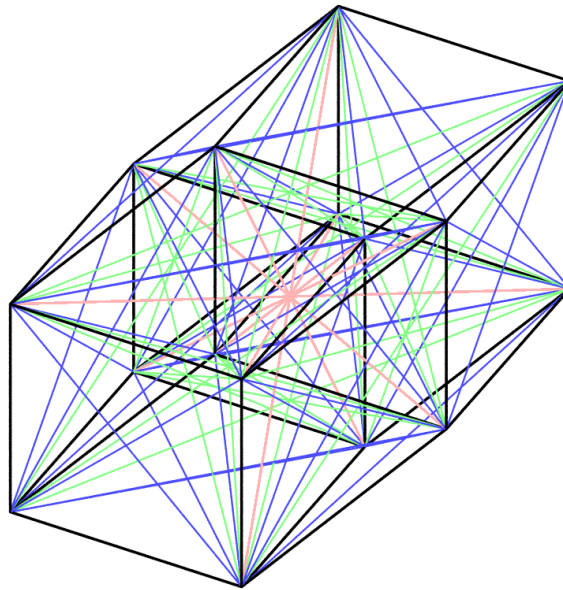
Figure 18 – 120 line segments connecting the vertices of a hypercube. The black segments
are the edges and other color segments are the diagonals
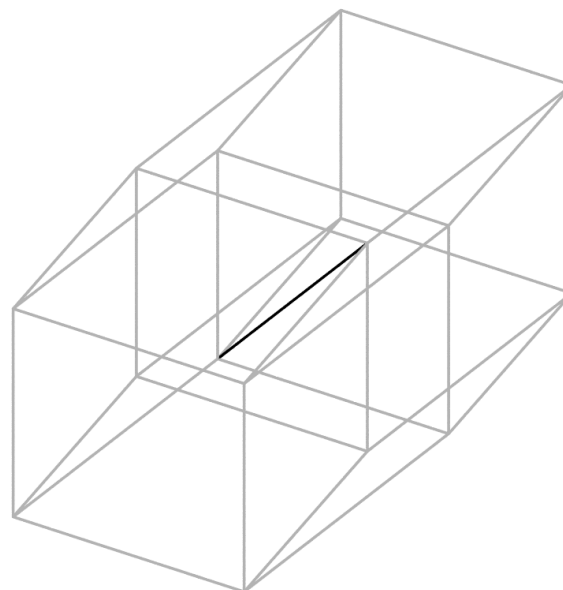
Figure 19 – The diagonal that is completely opposite to the direction of the vector $\vec{T}$ (black
line)

Therefore, by adding a tension material to this position, a tensegrity structure can
be obtained.

Then, consider the diagonal line that is added at the end. Among the line segments
connecting the vertices of a 4D polytope, the edges are referred to as 1D diagonal, the
diagonals included in a face but not classified as 1D diagonal are referred to as 2D
diagonal, the diagonals included in a solid but not classified as 2D diagonal or 1D

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diagonal are referred to as 3D diagonal, and the others are referred to as 4D diagonal. Let V_2, V_3, and V_4, be the number of vertices of a square, a cube, and a hypercube. Let E_2^1, E_3^1, and E_4^1 be the number of edges of a square, a cube, and a hypercube, because the number of edges is corresponding to the number of 1D diagonals. Let F_2, F_3, and F_4, be the number of faces of a square, a cube, and a hypercube, and Let C_2, C_3, and C_4, be the number of solid in a square, a cube, and a hypercube as well. In addition, let E_2^2, E_3^2, E_4^2, E_2^3, E_3^3, E_4^3, E_2^4, E_3^4, and E_4^4, be the number of 2D diagonals, 3D diagonals, and 4D diagonals in a square, a cube, and a hypercube, respectively. Then, V_i, E_i^1, F_i, C_i are given as follows:

$$V_i = 4 \times i \tag{17}$$

$$E_i^1 = E_{i-1}^1 \times 2 + V_{i-1} \tag{18}$$

$$F_i = F_{i-1} \times 2 + E_{i-1}^1 \tag{19}$$

$$C_i = C_{i-1} \times 2 + F_{i-1}^1 \tag{20}$$

In addition, E_i^2 and E_i^3 can be obtained as follows:

$$E_i^2 = 2 \times F_i \tag{21}$$

$$E_i^3 = 4 \times C_i \tag{22}$$

The total number of diagonals is the number of combinations that select two from V_i^1 vertices.

$$\sum_{j=1}^4 E_i^j = \frac{V_i \times V_{i-1}}{2} \tag{23}$$

From these equations, the value of E_i^4 can be obtained. Table 1 shows the number of vertices, diagonals, faces, solids in square, cube, and hypercube.

Table 1 – The number of vertices, diagonals, faces, solids in square, cube, and hypercube

	Vertices	1D diagonals	2D diagonals	3D diagonals	4D diagonal	Total diagonal	Faces	Solids
	D	E^1	E^2	E^3	E^4	$\sum E^i$	F	C
Square	4	4	2	0	0	6	1	0
Cube	8	12	12	4	0	28	6	1
Hypercube	12	32	48	32	8	120	24	8

As shown in this table, there are eight 4D diagonals of the hypercube. The diagonal line added at the end was belonging to 4D diagonal. Using the 4D diagonal line, the upper part and the lower part were connected with a tension material. The positional relationship between the two cubes is arbitrary as long as there is an overlapping part, however the distance between the two cubes should be the same as the length of the edge of the cube, so that the projection method was just corresponding to 4D to 3D cavalier projection among 4D to 3D oblique projection. Two cubes (the upper part and the lower part) were made of compression material, eight edges between them were connected by tension material, and a 4D diagonal was added by tension material as shown in Fig. 20.
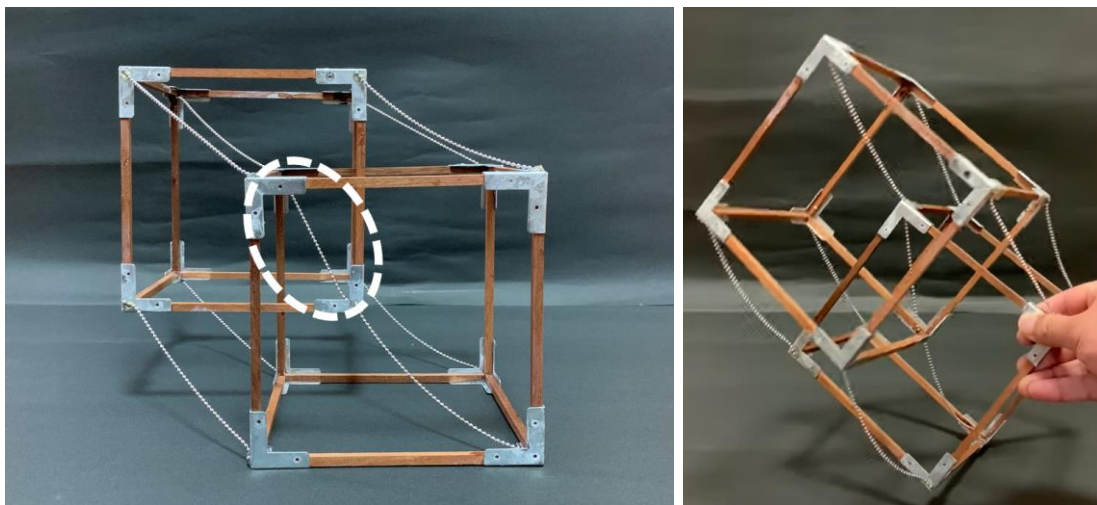

Figure 20 – The 3D models of hypercube projected by 4D to 3D oblique projection with tensegrity structure

The place surrounded by the dotted line in the figure is the place of the tension material added at the end. The upper part was completely floating in the air and is connected to the lower part only by the tension material. This structure of the form was the same even if the vertical and horizontal directions are exchanged, so the relationship between these the parts is the same even if the form is rotated as shown in Fig. 20 (right).

5   The Lampshade form based on 4D polytope with tensegrity structure

5.1   The lampshade form based on hyper cube

First, a lampshade using the hypercube tensegrity structure explained in the previous section was manufactured. Fig. 21 shows the dimensions of two cubes made of

compression members and the tension materials that connect corresponding vertices in the two cubes, and the pattern of the face.
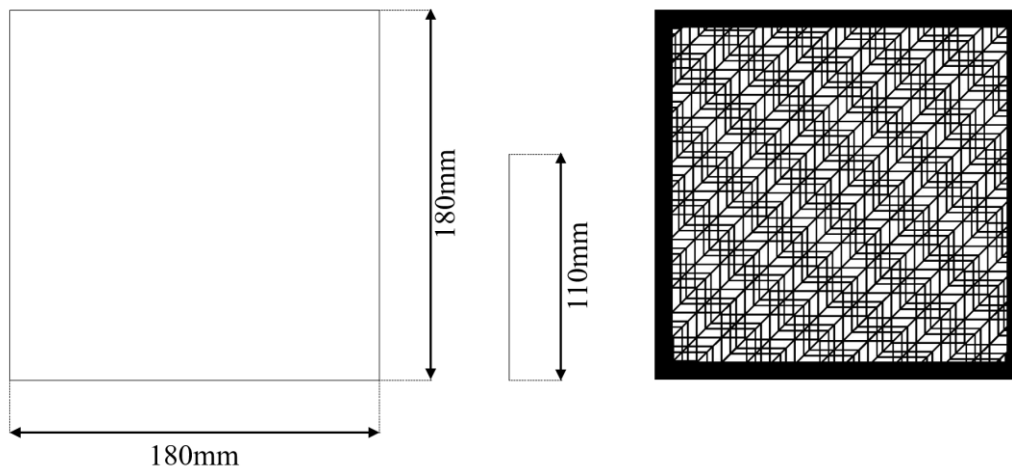

Figure 21 – The dimensions of two cubes made of a compression members and the tension material that forms the edge connecting the two cubes, and the pattern of the face. The dimension of the square on the cube (left) and the edge (center), and the pattern (right)

This pattern is a plane tessellation pattern composed of a hexagon, which is a 2D shape of a 3D model created by 4D to 3D oblique projection of a hypercube. A lampshade was assembled using these parts. Different from the form discussed in the previous section, it is necessary to attach a face to make it a lampshade, however a 3D projection model of a 4D form is a polyhedron containing a surface inside, so it is not suitable as a lampshade. Further, in this case, the faces penetrated by the edges restrict movement of the edge, so that an impression of floating cube cannot be obtained. Therefore, only the 6 faces that exist in the outermost part were attached to the form as shown in Fig. 22.
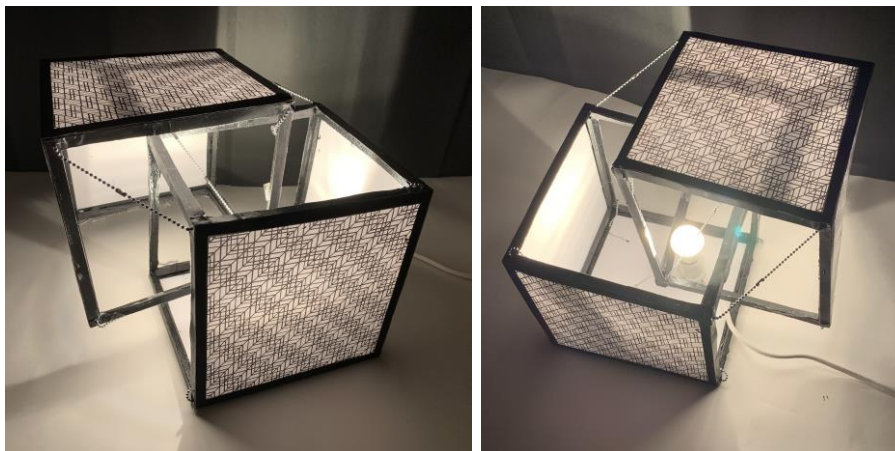

Figure 22 – The photos of the lampshade based on 4D to 3D projection of a hypercube

In this way, it plays a role of diffusing the light from the light source while maintaining a feeling of floating.

5.2 The lampshade form based on the polytope composed of quadrangular pyramids

Considering that a similar structure could be obtained by moving the polyhedron in parallel, two quadrangular pyramids were used to fabricate the lampshade. As shown in Fig. 23, quadrangular pyramids consisting of regular triangles with a side of 200 mm were used and these were arranged with a shift of 110 mm as shown in Fig.23.
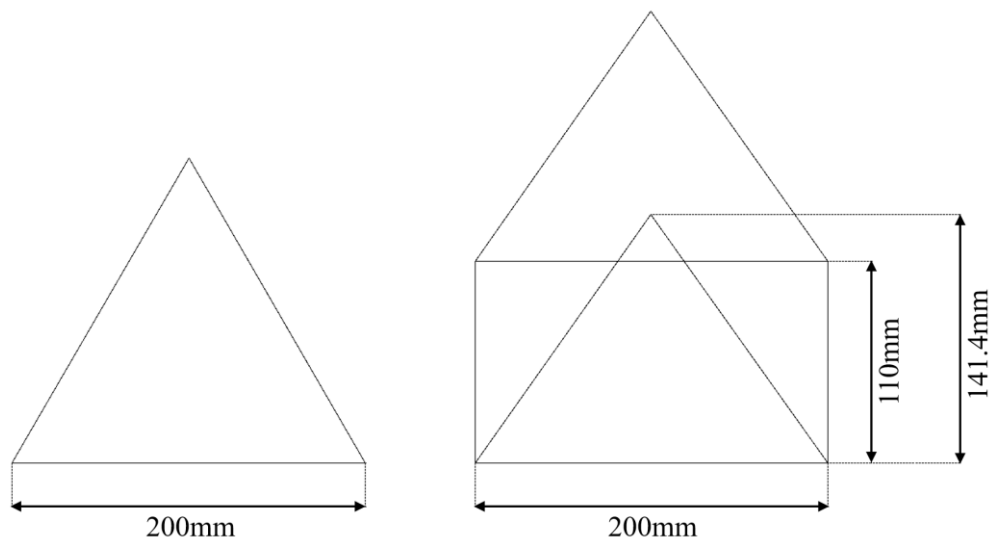

Figure 23 – The dimensions of quadrangular pyramids and structure of the lampshade composed of two quadrangular pyramids. The dimension of the regular triangle composed of quadrangular pyramids (left) orthographic drawing of the lampshade (right)

Let $\vec{T}$ (T_x, T_y, T_z) be the vector between two quadrangular pyramids in 3D projected space. In the poytope, there are $25 (= 5 \times 5)$ segment lines which connect the vertices of the quadrangular pyramids and that of the other quadrangular pyramids. In this case, as the values of T_x and T_z are not 0, it is impossible to obtain the appropriate vector D which is discussed in the previous section. Considering symmetry, four tension materials are added as shown in Fig. 24. In this case, the two faces were alternately attached on each quadrangular pyramid as shown. In many directions, the light from light source is diffused and the impression that the quadrangular pyramid is floating can be obtained.
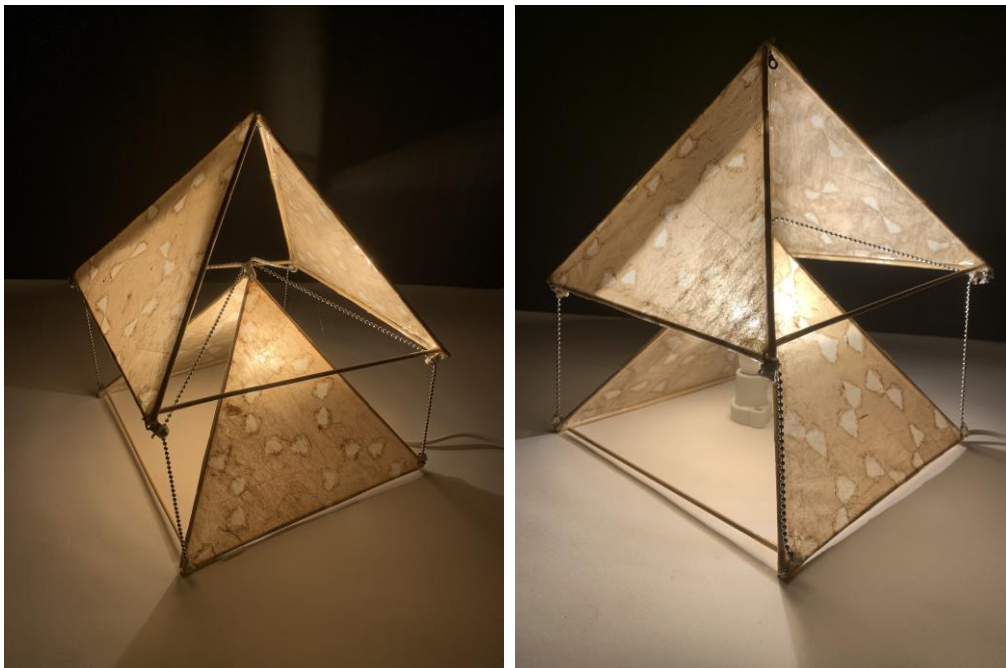
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Figure 24 – The photos of the lampshade based on the polytope composed of quadrangular pyramids

6 Summary and future works

In this paper, it was explained that geometric shapes have been used in the design of lampshades, and many examples of lampshade with geometric shapes were introduced. Then, the shape obtained by projecting a 4D form into 3D space was used as a tensegrity structure, and a method of utilizing the form in lampshade design was shown, and the actual examples of manufactured lampshades by the proposed method were introduced.

As the examples, basic 4D poytopes such as hypercube and polytope made of quadrangular pyramids were used. However, it is possible to use more complicated 4D polytopes and rotated simple polytope in the future. Though these examples were only evaluated from the point of view of design interest, consideration of the specific functions of these forms will be possible in the future, and these forms will be able to apply for devices of daylighting or other luminous devices.

Acknowledgement

References

ABE, Hirokazu. SAKAKI, Ai. SUZUKI, Hirotaka. HASHITERA, Tomoko. YASUFUKU, Kensuke. **Practical Descriptive Geometry**. Kyoritsu Shuppan Co., Ltd., 2020. (in Japanese)

MIURA, Koryo. **Proposition of Pseudo-Cylindrical Concave Polyhedral Shells.** Institute of Space and Aeronautical Science University of Tokyo Report No. 442, 141-163, 1969.

MIYAZAKI, Koji. **Polyhedral Encyclopedia.** MARUZEN Publishing, 2016 (in Japanese)

MIYAZAKI, Koji. **4D Shapes Encyclopedia.** MARUZEN Publishing, 2020 (in Japanese)

ODAKA, Naoki. SUZUKI, Hirotaka. KIRKEGAARD, Poul Henning. OSTERHAUS, Werner. OKAMURA, Atsumi. **Impression Evaluation of Paper Folding Lampshade By Danish Students**. Proc. of the 17th Intl. Conference on Geometry and Graphics, #166, 2016.

OKAMURA, Atsumi. ODAKA, Naoki. SUZUKI, Hirotaka. KIRKEGAARD, Poul Henning. OSTERHAUS, Werner. **Impression Evaluation of Paper Folding Lamp Shade by Japanese Students and Comparison with Results by Danish Students.** Proc. of the 17th Intl. Conference on Geometry and Graphics, #166, 2016.

SUZUKI, Hiroataka. **Application of Tangent Surfaces in the Design of Lampshades.** Proc. of the Fall Annual Conference of the Japan Society for Graphics Science,133-136, 2010. (in Japanese).

SUZUKI, Hirotaka. **Designing of Lighting Equipment Making Use of Tangent Surface and Control Method of the Surface by Hermite Curve.** Proc. of the 15th Intl. Conference on Geometry and Graphics, in CDROM, 2012.

SUZUKI, Hirotaka[a]. **A Generation Method of Developable Durface by Manipulation (Translation, Scaling and Rotation) of Curved Line and a Designing Method of Complicated Shapes by Combination of Generated Developable Durfaces.** Journal of Graphic Science of Japan, Vol. 48 Issue 1, 3-10, 2014. (in Japanese).

SUZUKI, Hirotaka[b]. **Control Method of Combined Developable Surface Design by Affine Transformation and Locus.** Proc. of the 16th Intl. Conference on Geometry and Graphics, in CDROM, 2014.

SUZUKI, Hirotaka. SAKAKI, Ai. YASUFUKU, Kensuke. and MATSUMOTO, Takashi. **Designing of Lampshade with 3D CG Application and Manufacturing of Designed Shape in Graphic Science Education.** Int. J. of Computer Applications in Technology, Vol.51, No.1, 9-14, (2015).

SUZUKI, Hirotaka. KIRKEGAARD, Poul Henning. ODAKA, Naoki. **Shape Analysis of PCCC Shell Structure by Geometrical Features –A Stable Shell Structure between Cylinder and PCCC Shell-.** Proc. of the 17th Intl. Conference on Geometry and Graphics, #100, 2016.

SUZUKI, Hirotaka. KOYANAGI, Nao. SAKAGAMI, Kimihiro. **Combination of Non-Flat Pattern and Geometrical Pattern by Transmittance for Lampshade Design.** Proc. of the 11th Asian Forum on Graphic Science, P08, 2017.

SUZUKI, Hirotaka[a]. **Utilization of Plane Tessellation for Paper Folding and Geometrical Transmittance Pattern for Lampshade Design.** Proc. of the 18th Intl. Conference on Geometry and Graphics, pp.1075-1083, 2018.

SUZUKI, Hirotaka[b]. **Proposal of Geometrical Lampshade Design Method Making Use of Wave Shape Tessellation and Non-Flat Texture**. Proc. of the Asia Lighting Conference 2018, 2018.

SUZUKI, Hirotaka. **Proposal of the Method Generating Layered Plane Tessellations Making Use of Dissection. -Lampshade Design with Non-Flat Texture and Transmittance Texture-.** Proc. of the Asia Light-ing Conference 2019, 2019.

SUZUKI, Hirotaka. **Utilization of Nonconvex Shapes as Horizontal Section of Paper Folding Lampshade Constructed with Skew Quadrilateral Elastic Folding.** Proc. of the 19th Intl. Conference on Geometry and Graphics, pp.458-470, 2020.

YOSHIMURA, Yoshimaru. **On the Mechanism of Buckling of a Circular Cylindrical Shell under Axial Compression**. Reports of the Institute of Science and Technology (The University of Tokyo), Vol. 5, No.5, 179-198, 1951. (in Japanese).